Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced Level

Friday 23 June 2006 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. (*a*) Prove, using integration, that the moment of inertia of a uniform rod, of mass m and length $2a$, about an axis perpendicular to the rod through one end is $\frac{4}{3}ma^2$.

(3)

(*b*) Hence, or otherwise, find the moment of inertia of a uniform square lamina, of mass M and side $2a$, about an axis through one corner and perpendicular to the plane of the lamina.

(3)

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2. A particle of mass 0.5 kg is at rest at the point with position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$ m. The particle is then acted upon by two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$. These are the only two forces acting on the particle. Subsequently, the particle passes through the point with position vector $(4\mathbf{i} + 5\mathbf{j} - 5\mathbf{k})$ m with speed 12 m s^{-1}. Given that $\mathbf{F}_1 = (\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ N, find $\mathbf{F}_2$.

(9)

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3. A particle P moves in the x-y plane and has position vector $\mathbf{r}$ metres at time t seconds. It is given that $\mathbf{r}$ satisfies the differential equation

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} = 2\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}.$$

When $t = 0$, P is at the point with position vector $3\mathbf{i}$ metres and is moving with velocity $\mathbf{j}$ m s^{-1}.

(*a*) Find $\mathbf{r}$ in terms of t.

(8)

(*b*) Describe the path of P, giving its cartesian equation.

(2)

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4. A force system consists of three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a rigid body.

$\mathbf{F}_1 = (\mathbf{i} + 2\mathbf{j})$ N and acts at the point with position vector $(-\mathbf{i} + 4\mathbf{j})$ m.
$\mathbf{F}_2 = (-\mathbf{j} + \mathbf{k})$ N and acts at the point with position vector $(2\mathbf{i} + \mathbf{j} + \mathbf{k})$ m.
$\mathbf{F}_3 = (3\mathbf{i} - \mathbf{j} + \mathbf{k})$ N and acts at the point with position vector $(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ m.
It is given that this system can be reduced to a single force $\mathbf{R}$.

(*a*) Find $\mathbf{R}$.

(2)

(*b*) Find a vector equation of the line of action of $\mathbf{R}$, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a parameter.

(10)

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5. A space-ship is moving in a straight line in deep space and needs to reduce its speed from U to V. This is done by ejecting fuel from the front of the space-ship at a constant speed k relative to the space-ship. When the speed of the space-ship is v, its mass is m.

(*a*) Show that, while the space-ship is ejecting fuel, $\frac{\mathrm{d}m}{\mathrm{d}v} = \frac{m}{k}$.

(6)

The initial mass of the space-ship is M.

(*b*) Find, in terms of U, V, k and M, the amount of fuel which needs to be used to reduce the speed of the space-ship from U to V.

(6)

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6. A uniform circular disc, of mass m, radius a and centre O, is free to rotate in a vertical plane about a fixed smooth horizontal axis. The axis passes through the mid-point A of a radius of the disc.

(*a*) Find an equation of motion for the disc when the line AO makes an angle θ with the downward vertical through A.

(5)

(*b*) Hence find the period of small oscillations of the disc about its position of stable equilibrium.

(2)

When the line AO makes an angle θ with the downward vertical through A, the force acting on the disc at A is $\mathbf{F}$.

(*c*) Find the magnitude of the component of $\mathbf{F}$ perpendicular to AO.

(5)

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7. Particles P and Q have mass $3m$ and m respectively. Particle P is attached to one end of a light inextensible string and Q is attached to the other end. The string passes over a circular pulley which can freely rotate in a vertical plane about a fixed horizontal axis through its centre O. The pulley is modelled as a uniform circular disc of mass $2m$ and radius a. The pulley is sufficiently rough to prevent the string slipping. The system is at rest with the string taut. A third particle R of mass m falls freely under gravity from rest for a distance a before striking and adhering to Q. Immediately before R strikes Q, particles P and Q are at rest with the string taut.

(*a*) Show that, immediately after R strikes Q, the angular speed of the pulley is $\frac{1}{3}\sqrt{\left(\frac{g}{2a}\right)}$.

(5)

When R strikes Q, there is an impulse in the string attached to Q.

(*b*) Find the magnitude of this impulse.

(3)

Given that P does not hit the pulley,

(*c*) find the distance that P moves upwards before first coming to instantaneous rest.

(6)

TOTAL FOR PAPER: 75 MARKS

END